A Review of Methods for the Geometric Post-Processing of Topology Optimized Models

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Abstract: Topology optimization (TO) has rapidly evolved from an academic exercise into an exciting discipline with numerous industrial applications. Various TO algorithms have been established, and several commercial TO software packages are now available. However, a major challenge in TO is the post-processing of the optimized models for downstream applications. Typically, optimal topologies generated by TO are faceted (triangulated) models, extracted from an underlying finite element mesh. These triangulated models are dense, of poor quality, and lack feature/parametric control. This poses serious challenges to downstream applications such as prototyping/testing, design validation, and design exploration.

One strategy to address this issue is to directly impose downstream requirements as constraints in the TO algorithm. However, this not only restricts the design space, it may even lead to TO failure. Separation of post-processing from TO is more robust and flexible. The objective of this paper is to provide a critical review of various post-processing methods, and categorize them based both on targeted applications, and underlying strategies. The paper concludes with unresolved challenges and future work.

1 Introduction

Various design optimization methods are used today to solve engineering problems; these include size, shape and topology optimization. The focus of this paper is on topology optimization [1–3], that often serves as a starting point for size and shape optimization. Topology optimization (TO) has rapidly evolved from an academic exercise into an exciting discipline with numerous industrial applications. Popular applications include optimization of aerospace and aircraft components [4–9], automotive components [10–12], biomedical devices [13–17] structure design [18–22], compliant mechanisms [23–26], thermofluid applications [27–34], etc

To illustrate the concepts behind TO, consider the structural problem posed in Figure 1 where the objective is to find the stiffest topology, i.e., topology with the lowest compliance, within the given design-space with 50% volume fraction.

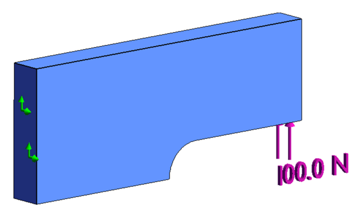

Figure 1: *A structural problem over a design space.*

This can be solved rapidly today via any of the well-known TO methods [35–44]. A typical optimized topology is illustrated in Figure 2.

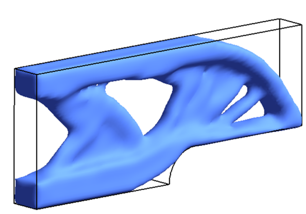

Figure 2: *An optimized topology.*

Rapid generation of such optimized designs is particularly beneficial during the early stages of the design process. However, one of the drawbacks of TO is that the optimal topology, such as the one in Figure 2, is typically extracted as a *faceted (triangulated)*

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model, from the underlying finite element mesh, independent of the specific TO method. This extraction relies on classic isosurface methods such as marching cubes [45]; see Figure 3.

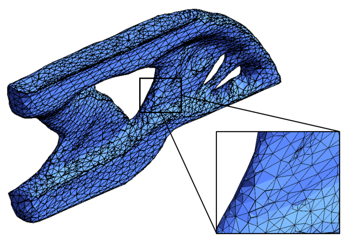


Figure 3: *The faceted representation with noisy and poor quality triangles.*

The faceted models are often of poor quality, non-smooth, dense and lack feature/parametric control. For example, the faceted model in Figure 3 contains over 25,000 triangles, where, most of them are of poor quality. This is often exacerbated in real-world problems. As an illustration, for the TO challenge problem posed during 2019 Topology Optimization Roundtable Conference, Albuquerque [46], millions of elements are necessary to capture critical features. This results in faceted model with millions of triangles (see Figure 4). Such triangulated models are ill-suited for downstream applications such as prototyping/testing, design validation, and design exploration.

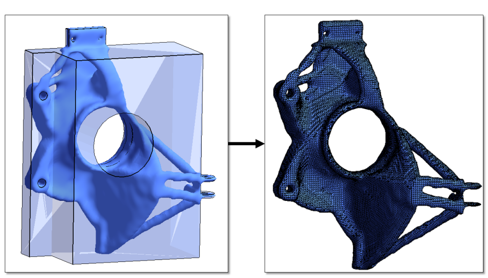


Figure 4: *A TO model with millions of triangles.*

Most TO commercial packages do not have automated tools for post-processing. Post-processing is loosely defined here as the process of converting the faceted TO models into other geometric representations that are more suitable for various downstream applications. Such geometric representations include skeletal representation, simplified triangulated model, NURBS-representation, volume decomposition and so on. Thus post-processing strategies can range from simple remeshing, to extraction of skeleton, and fitting of analytic surfaces. Some of the early commercial packages relied on manual tracing of the TO model for reconstruction, i.e., the faceted models are superimposed over the design space, and the geometry is reconstructed via sketching and Boolean operations. This is laborious and error-prone. However, some commercial systems are beginning to support post-processing with various degree of success. The most common strategy used in commercial systems is surface based reconstruction (see Section 4 for a description). PTC Creo® uses subdivision technique, while Evolve® and Rhino®, MeshMixer® use Non-Uniform Rational B-Splines (NURBS) based reconstruction. Fusion 360 Generative Design relies on T-splines to generate multiple watertight CAD models that satisfy designer's requirements. None of these tools efficiently generate a parametric feature based CAD model that meets all downstream requirements discussed in the subsequent section.

A survey was conducted among users of a free topology optimization service (cloudtopopt.com) [47], sponsored by the National Science Foundation (www.nsf.gov). One of the questions posed to the users was: *Rank what would you like topology optimization software to include in order of preference?* Five specific choices were provided, with one open choice. Among the 85 responses received, 49% choose: *Generate feature-based CAD model of the optimized design*; see Figure 5. Lack of automated tools for model reconstruction can be a serious detriment to broader acceptance and proliferation of TO.

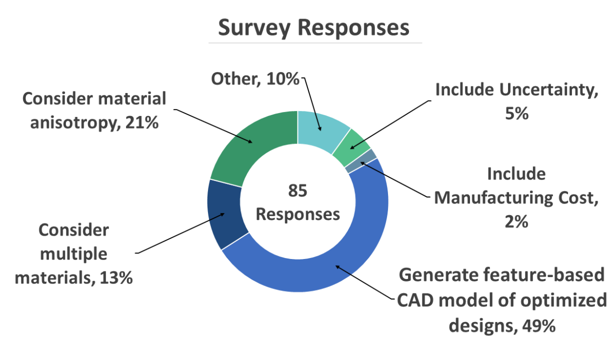


Figure 5: *Results from a survey of TO users.*

Researchers have proposed several strategies and methods to address this challenge. Prior to discussing these strategies, we consider three important downstream applications in Section 2, and summarize their requirements. Then, in Section 3 we consider proposed methods that attempt to meet these downstream requirements by directly incorporating them as constraints in the TO algorithm. These direct methods, however, have limitations. In Section 4, we consider post-processing methods that rely on a combination of design rules and computational algorithms. For pedagogical reasons, these are further categorized based on the underlying dimension. Conclusions and future work are discussed in Section 5.

2 Downstream Applications

In this section, we consider three representative downstream applications, namely, *prototyping, validation and (design) exploration*, as illustrated in Table 1. These three applications are representative and not exhaustive. Further, since the requirements for these applications overlap, these are best represented via a Venn diagram as in Figure 6. For example, "feature control" is essential for design exploration, but not necessary for validation and prototyping. However, "retaining critical features" is essential for all three applications. These requirements are further elaborated below, and will be used later to evaluate different post-processing methods and strategies.

2.1 Prototyping

The simplest downstream application is prototyping and testing; the objective is to fabricate the TO model for testing, inspection and evaluation. A primary requirement is that critical features, edges and surfaces must be retained for repeatable testing. For example, if a load is applied on a cylindrical feature in the initial design, then this surface will be critical for prototyping and testing. Secondly, non-critical surfaces must be smooth, both for aesthetic and testing purposes. Finally, the recovered model must meet the constraints of the fabrication process. For example, for conventional milling, tool accessibility is important; for certain additive manufacturing processes, overhang surfaces must be avoided, and so on. However, parametric representation of the model, for example, is not critical for prototyping.

2.2 Design Validation

The second critical application is design validation where the TO model must be validated through analysis methods such as finite element analysis (FEA). FEA models used within TO are often vastly simplified, for example, they often rely non-conforming voxel mesh to accelerate FEA. To support rigorous FEA-based design validation, retaining critical features is once again important. In addition, one must be able to create a high-quality mesh that conforms to critical surfaces and features. This is more stringent than smoothness requirements for prototyping. Specifically, the recovered model should not contain sharp geometric features that could lead to erroneous simulation results. Finally, the reconstructed model must be functionally equivalent to the TO model in that the behavior of the reconstructed model should not differ significantly from that of the TO model.

2.3 Design Exploration

The final application, and often the lofty goal, is design exploration and productization. This is the most demanding since the reconstructed model must be (easily) editable by the designer to meet various functional and manufacturing constraints. The model must allow parametric changes (example: increasing thickness of a strut), suppression/inclusion of features, and be compatible with popular computer-aided-design (CAD) packages.

3 Constrained Optimization

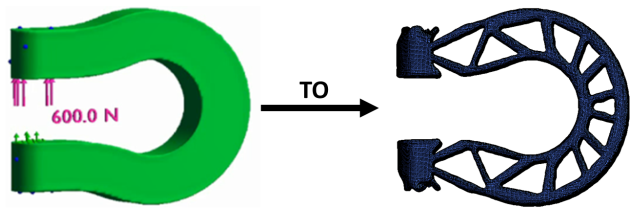
Although the objective of this paper is to survey post-processing methods, we briefly review strategies for imposing downstream requirements directly as constraints within the TO algorithm; This serves two purposes: (1) if the downstream requirements are sufficiently simple, a constraint based TO may be sufficient, (2) to highlight the deficiencies of constraint based strategies.

Researchers have largely focused on including prototyping and design exploration requirements in TO. We are not aware of strategies to incorporate validation/analysis requirements (ex: high-quality surface mesh) into TO. However, due to the overlap in requirements, many of the techniques discussed below can directly assist in efficient validation. The reader is referred to [48] for a broader discussion on constrained based TO.

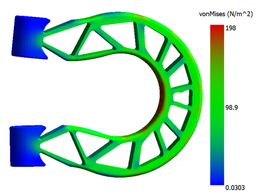
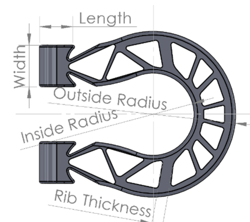
3.1 Prototyping Constrained TO

Researchers have proposed several methods to incorporate prototyping, i.e., manufacturing, constraints directly into TO to minimize post-processing. Harzheim and Graf [49], [50] provide a review of early work on TO for cast parts. Liu and Ma [51] present a more recent survey on manufacturing focused TO. Zuo et al. [52] incorporated machining constraints, while Li et al. [53] imposed extrusion constraints, and Lui et al. [54] have explored symmetry and pattern repetition constraints in topology optimization. Li et al. [55] incorporated multi-directional molding constraints in TO for cast parts. Vatanabe et al. [56] incorporated constraints such as minimum size, symmetry, extrusion, turning, casting, forging and rolling into the optimization.

Table 1: *Typical downstream applications of topology optimized designs.*



Topology optimization of a clevis model.

Prototyping and Testing	**Design Validation**	**Design Exploration**
		

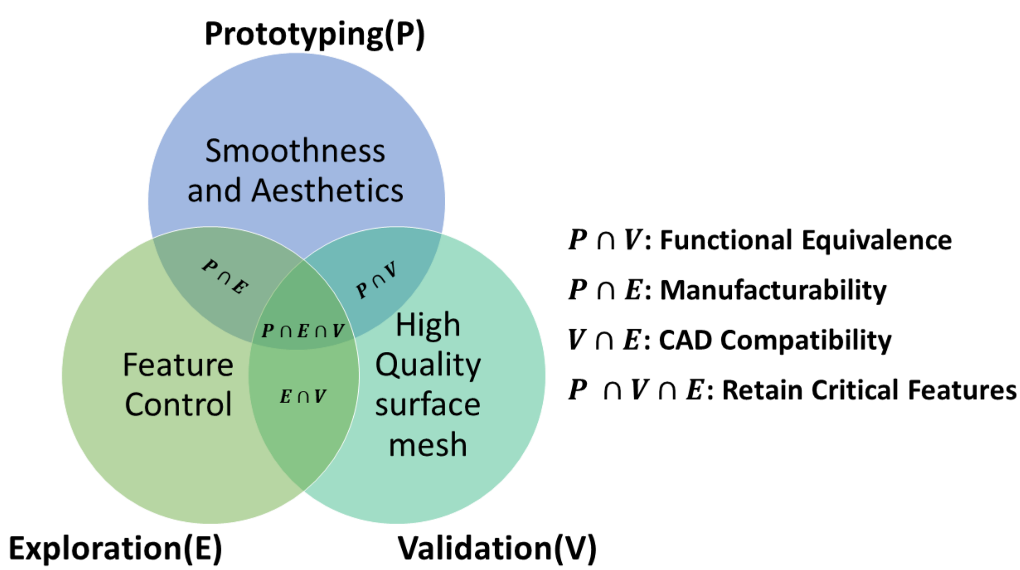


Figure 6: *Requirements of model for different downstream applications*

Lui and Ma [57] performed least-square fitting of 2.5D and 3D machining-based features over the evolving boundary, while Groen and Sigmund [58] used homogenization method for generating manufacturable microstructure based designs. Amir et al. [59] proposed an approach for simultaneously satisfying physics based constraints (compliance, volume) as well as kinematics based constraints (manufacturing, accessibility). There has been significant interest recently in incorporating additive manufacturing (AM) [60] constraints in TO [61]. Doutre et al. [62] compare existing state-of-art tools to obtain CAD models from TO, specifically for AM. Lui and To [63] have used feature fitting on the TO design for additive manufacturing. Leary et al. [64] identify boundaries that require supports in additive manufacturing; these boundaries were then modified to generate support-free structures. Amir and Suresh [65] used topological sensitivity to incorporate AM support structure constraints in TO. Similarly, Mass and Amir [66], and Garaigordobil et al. [67, 68] incorporated overhang constraints.

A minimum-member-size for additive manufacturing has been used as a constraint in TO by Kwok et. al. [69]. Thin features and volume of support structures have been added as constraints by Mhapsekar et. al [70]. Qian [71] added undercut control and minimal overhang angle as constraints in SIMP based TO.

Similarly, Mezzadri et al. [72] and Matthijs [73] designed self-supporting support structures using TO for additive manufacturing of parts. Chandrasekhar et al. [74] proposed a methodology to incorporate build direction, and fiber orientation into a TO formulation for short fiber reinforced polymers components. Stuben et.al [75] use multiscale TO to generate 2D designs for additive manufacturing. See Lui et al. [76] for an extensive review on TO for AM.

3.2 Design Exploration Constrained TO

Next we consider strategies to include design exploration requirements into TO. Bendsoe and Rodrigues [77] explored the idea of using TO models as a precursor to shape optimization in 2D. Olhoff N. [78] was one of the earliest to propose CAD-integrated TO to reduce design lead time. Zhou and Wang [79] combined CSG with topology/shape optimization to generate free-form geometric designs. Chen et. al [80] proposed a B-spline based method for combined shape and topology optimization. Tang and Chang [81] presented an integrated approach to combine topology optimization and shape optimization using B-splines to represent the boundaries. Lin et al. [82] used image processing to convert the gray-scale results of TO to obtain a parametric geometry in 2D. Zhang and Kwok [83] performed TO over a parametrized 2D mesh obtained by mapping a 3D domain onto a 2D domain. The optimized results are then mapped back to obtain a 3D geometry. Similarly, Christiansen et al. [84] combined shape and topology optimization for 3D structures using explicit shape representation.

Another popular strategy to support design exploration is to directly incorporate design features during TO. Guo et al. [85], Zhang et al. [86], [87] have used moving morphable components to represent the boundaries of TO designs. The size, shape, and orientations of these components are used as variables during topology optimization to generate designs with predefined features. Bell et al. [88] and Norato et al. [89] used parametrically-defined bars, while Zhang et al. [86] used parametrically-defined bars and plates to obtain TO designs. Lin et al. [90] used NURBS to represent the boundary of features arising during TO. Holes represented by NURBS are inserted in the design domain and their control points are used as design variables to generate parametrically-defined TO geometry. Gao et al. [91] replaced discrete density field by NURBS and then imposed user defined geometric constraints during topology optimization of beams and plates. Zhang et al. [92] traced the topological changes in the geometry using B-Splines to construct free-form shapes. Norato [93] used union of 2D super-shapes to generate free-form geometry.

Da et al. [94] used bi-directional evolutionary structural optimization (BESO) with level set function to generate results with smooth boundaries. Jahangiry et. al [95], Kang et al. [96], Seo et al. [97] and more recently, Gai et al. [98] have used spline based isogeometric analysis for Topology Optimization. Gao et al. [99] have used density distribution function (DDF) for isogeometric Topology optimization to obtain smooth NURBS surface in 2D and 3D.

More recently, machine learning algorithms have been applied towards post-processing of TO models. For example, Sosnovik and Oseledets [100] trained their neural network using image segmentation to obtain final designs from intermediate results of TO, thereby reducing the computational effort. Shen and Chen [101] and Rawat and Shen [102, 103] proposed a conditional generative adversarial network (GAN) to incorporate design constraints such as minimum radius in TO of planar structures. Lei et. al [104] used support vector regression (SVR) and K-nearest-neighbour (KNN) models to predict topology optimized designs.

3.3 Benefits and Limitations

Adding downstream constraints directly into TO eliminate the need for expensive post-processing. Indeed, this may be a practical and viable option in simple scenarios. However, there are several limitations to these strategies:

1. *Reduced design space:* Adding constraints necessarily reduces the design space, and consequently, the performance of the optimized design.

2. *Computational challenge:* Adding constraints can significantly increase the cost of TO; further, the optimization may even fail if improper constraints are imposed.

3. *Lack of generality:* The strategies are often limited in scope; for example, the extension of feature-based strategy to 3D is an open challenge, and not all manufacturing processes can be imposed as a constraint. Further, most methods involve manual intervention and expertise to generate the CAD geometry.

4. *Lack of flexibility:* Finally, since constraint-based strategies often target a particular application, exploring other options is often not viable once the optimization is complete.

Thus, one must resort to post-processing of TO models, and this is discussed next.

4 Post-Processing Strategies

As one can expect, different post-processing strategies fulfill different requirements. For example, if the downstream application is finite element analysis, then post-processing the surface mesh, while imposing geometric and quality constraints may be sufficient. On the other hand, for design exploration, recreating a CAD-compatible parametric model will be necessary, and so on.

Post-processing strategies can be classified based on the underlying dimension as in Table 2. Specifically, if the post-processing is based on first extracting a lower-dimensional skeleton, it is classified as 1D. If the strategy relies directly on post-processing the triangulated surface, it is classified as 2D. Finally, if the strategy relies on volume decomposition of the TO model, it is classified as 3D. Similar classification strategies have been proposed by Fabio [105] for reconstruction of geometry from cloud data points and by Thakur et al. [106] for CAD model simplification. As stated earlier, skeleton based post-processing is largely limited to thin beam-like TO designs. In addition, two recurring challenges here are: (1) robust handling of junctions where skeletal branches meet, and (2) extraction of cross-sections.

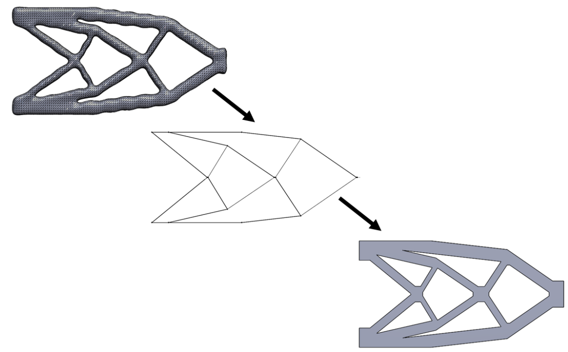


Figure 7: *Geometry reconstruction using skeleton.*

4.1 Surface Based (2D)

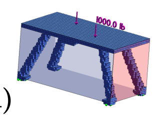
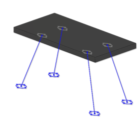
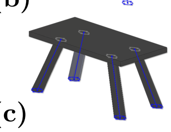
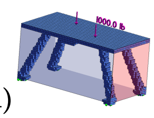
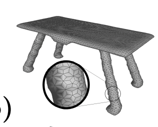
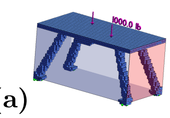
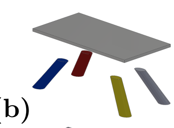
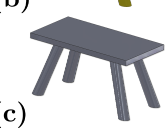
The second, and probably the most common, category of post-processing is surface reconstruction. There are three fundamentally different surface-based methods: *remeshing, sub-division, and surface-fitting*. In remeshing, one directly creates an improved triangulation from TO triangulation. In subdivision, a predefined set of rules are used to recreate a discretized surface (triangles and quads) that best fits the original surface. Finally, in surface-fitting, the triangulation is replaced by a parametric surface (such as NURBS) or analytical surface (such as a cylinder).

4.1.1 Remeshing

Remeshing creates an improved triangulation from a potentially noisy triangulation, or sampled (scanned) data [107]. There are two popular methods of remeshing: *implicit and explicit*, and there are several implementations; for example, see PMP [108] and Instant-Meshes [109].

Implicit remeshing methods rely on constructing a smooth scalar field from the input triangulation; the scalar field is then used to recreate a high-quality re-triangulation. For example, Kazdhan [110] proposed the Poisson reconstruction method to generate water-tight meshes. Implicit methods often result in undesirable smoothening of sharp edges. Attene et al. [111] proposed an edge-sharpener algorithm while Nielson et al. [112] used dual marching cubes to recover shape features from the triangulated models. Thomos et al. [113] modified marching cubes tables for topological guarantees. Although implicit methods are robust, numerically stable and generate water-tight models, they can be computationally expensive, and are non-local, i.e., small defects in one region can affect the triangulation globally.

Table 2: *Proposed classification of post-processing strategies*

Classification	**Skeletal (1D)**	**Surface (2D)**	**Volume(3D)**
Underlying technique	Reconstruction via skeleton	Surface fitting and/or mesh simplification	Volume decompostion and approximation
Reconstruction process	**(a)**  **(b)**  **(c)** 	**(a)**  **(b)**  **(c)** 	**(a)**  **(b)**  **(c)** 
Strengths	Well suited for beamlike models	Relies on popular remeshing methods	Ideal for suppressing small features
	Applicable to all downstream applications	Applicable to all TO models	Easy to retain critical features
Weaknesses	Handling of junctures	Stitching of gaps, and retaining sharp features	Not suited for complex TO models
	Not suitable for all TO models	Automation	Automation

Explicit remeshing methods often rely on Delaunay triangulation of point data [114], [115]. Dey and Goswami [116] proposed a water-tight remeshing algorithm. Explicit methods are local, and easy to implement but are less stable [117]. Figure 8 illustrates remeshing of triangulated surface into a triangular/quad mesh. This reconstruction was performed using Poisson surface reconstruction [110] implemented in Meshlab® v2016.12 ; the processed geometry is smoother and contains a fewer number of triangles/quads.

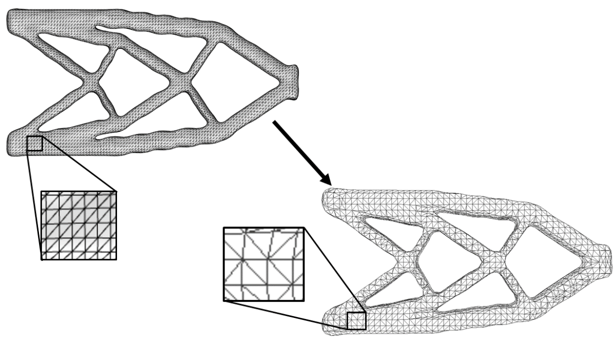


Figure 8: *Remeshing of triangular meshes using screened Poisson surface reconstruction*

4.1.2 Fitting

The objective of surface fitting is to replace the triangulation with either analytical primitives such as planes, spheres, cylinders, etc. or parametric surfaces such as NURBS. The techniques discussed below are often used in the context of scanned data [118] but directly apply to TO post-processing (especially parametric surface fitting). Figure 9 demonstrates smoothing and fitting of the TO model using NURBS. The fitting was performed using Rhino® 6, released in February 2018. Control points generated through surface fitting provide local control over the surface.

Fitting primitives only applies when the underlying surface is analytic. Several methods have been proposed to fit analytic surfaces. Li et al. G. Yi, B. D. Youn, and N. H. Kim [119] fit basic geometric features such as lines, arcs, circles, fillets, extrusion and sweep on boundary extracted from a topology optimized design. [120] proposed Globfit algorithm to recover a set of locally fitted primitives. Schnabel [121] proposed an efficient RANSAC algorithm to recover analytic shapes from noisy input models.

In parametric surface fitting, NURBS are often used to fit the triangulation. Joshi,et al. [122] created an open source tool that fits a NURBS surface over the mesh using least square fitting. Non-design features are then added manually to the resulting surfaces. Continuity between multiple patches was not discussed. Lui et al. [123] used adaptive B-spline fitting of the surface. The resulting geometry is a smooth parametric model suitable for further shape optimization and targeted for additive manufacturing. Chacon et al. [124] developed a software tool that fits B-Splines on the boundaries of 2D Topology optimized designs and converts them to IGES format for CAD compatibility.

Koguchi and Kikuchi [125] used marching cube based iso-surface extraction algorithm to construct biquartic surface splines. The parametric model preserves all critical features such as flat surfaces and sharp edges. The resulting geometries require further processing to make them manufacturable.

Marsan and Dutta [126], extracted smooth contours layer-by-layer. These contours are then used to fit spline surfaces with C1 continuity. This method works for post-processing of models with holes/branches, but it fails to retain critical features and surfaces. Yoely, et al. [127] use B-splines to represent the boundaries of topology optimized designs for generating parametric 2D geometries. Similarly, W. Zhang, L. , T. Gao, and S. Cai [92] make use of closed B-Splines curves to trace optimum topology in 2D geometries.

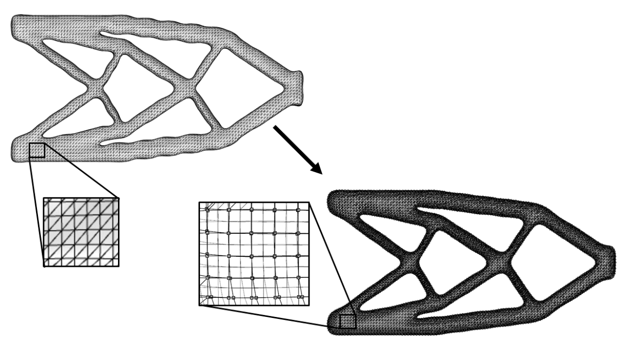


Figure 9: *NURBS surface fitting with control points*

A common challenge in surface fitting are gaps between surfaces. Various hole-filling approaches have been proposed. Zhao et al. [128] proposed an advancing front method. Branch et al. [129] used a local radial basis function to fill the space with B-spline surfaces. Curless et al. [130] used volumetric diffusion method to fill gaps. Liepa [131] combined remeshing and fairing method to smoothly bridge surface meshes.

4.1.3 Subdivision

Subdivision surfaces were introduced as an alternative to NURBS modeling. A subdivision surface is a representation of smooth surface over a piece wise linear polygon mesh similar to Bezier curve in 2D. A smooth surface is achieved by iterative subdivision scheme, defined by a set of rules. Geometry reconstruction based on subdivision surfaces is illustrated in Figure 10 using PTC Creo® 6.0.1.0. The subdivision is semi-automated and the surface maintains connectivity with non-design features, while retaining critical surfaces and edges.

Catmull-Clark subdivision [132] creates new vertex points using the face points and edge points. These new vertex points are then connected for each quadruple to create new face quadrilaterals. Though this method generates aesthetically pleasing surfaces, planar surfaces are often destroyed.

Doo-Sabin [133] subdivision surfaces are created by replacing each vertex with face. The new faces created at the vertices are not necessarily planar. Few other subdivision based surface generation methods include Loop [134], mid-edge subdivision [135]. Subdivision surfaces offer a high level of user control, and can reproduce sharp edges and corners. Despite these advantages, maintaining second-order behavior near singularities is a major challenge for subdivision surfaces, and for complex shapes, it is almost impossible to remove mesh singularities.

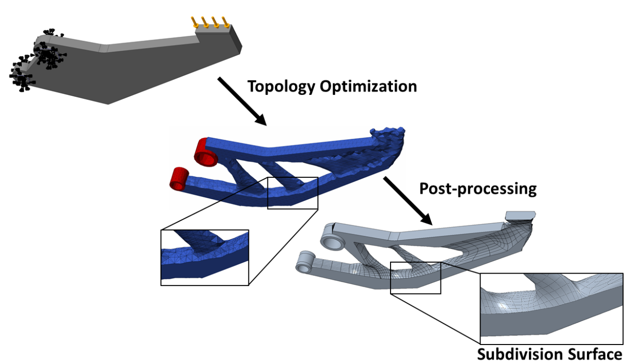


Figure 10: *Geometry reconstruction on TO design with sub-division.*

Marinov et al. [136] recently used non-uniform rational Catmull-Clark (NURCC) surfaces [137] to convert generative design models to editable B-rep models. The triangular mesh is separated out from the non-design solids and is approximated via NURCC surfaces. Replacing triangular meshes with quad mesh makes it easier for local editing of shapes. Non-design solid geometries are then merged with the NURCC surfaces to construct watertight models. Although the authors use generative design, the same concept could be applied to TO models. This is a significant step towards the automated generation of parametric CAD geometry from TO in product design workflow.

4.2 Volume Based (3D)

The primary idea in volume based post-processing is to reconstruct the model through volume decomposition, and Boolean operations. For example, Hsu an Hsu [138], Shu, et al. [139], extract representative cross sections from the topology optimized designs. The boundary points are used as control points to create B-spline boundary curves. Parametric 3D solids are created in a CAD using sweeps through these boundary curves. This method fails if there is a significant difference in the shape/topology between two successive boundary curves.

Cuillière, et al. [140, 141] separate out the non-design from the design domain. The optimized design is then merged with the non-design features to obtain the final geometry. This method retains critical features from the initial geometry. Connectivity between design and non-design features is a challenge since they are highly dependent on the mesh size. Further, due the use of unstructured mesh, symmetry is lost in the optimized design. Larsen and Jensen [142] used 2D shape template fitting to create sweep geometries. These 3D solid bodies constructed using sweep are subtracted from the initial design domain. The algorithm requires manual intervention to fit different shapes. Recently Du. el.al [143] proposed InverseCSG algorithm to convert 3D models to CSG trees.

The methods discussed above work directly on the TO models. Alternately, one can also work with the voids (negative space) as illustrated in Figure 11. This approach is preferable if the negative components are simpler to approximate than the full TO design. Further, critical features can be easily retained. This post-processing strategy on topology optimized designs is currently being developed as a research tool within Pareto [40].

Volume based methods are effective only if the TO design can be decomposed into simpler sweep-representable volumes. Further, automatic identification of source/target profiles and sweep path is non-trivial.

5 Conclusions

Topology optimization continues to grow in importance, and is being increasingly adopted by the industry to accelerate design. However, one of the roadblocks is the efficient and automated post-processing of topology optimized models for various downstream applications. In this paper, we identified three major applications and their requirements. For simple designs, it may be possible to include downstream re-

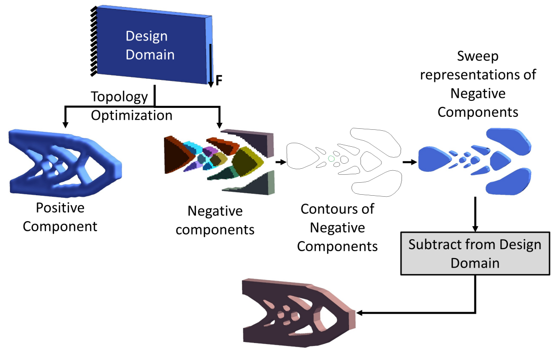


Figure 11: *A classical cantilever beam topology optimization problem with geometry reconstruction.*

quirements as constraints in topology optimization. However, in more complex scenarios, post processing is unavoidable. Various post-processing strategies were reviewed, and classified based on the implicit dimension.

It is evident that research gaps remain. In skeletal based (1D) methods, computing the cross-section, merging of skeletal branches and handling of pathological cases require significant manual intervention. In addition, skeletal methods largely apply to tubular models. Surface based (2D) methods are the most advanced and promising. Among them, triangle-to-quad mesh conversion is the most popular since quad meshes are easier to edit. However, in practice, editing of quad-meshes requires carefully defined geometric constraints. Other challenges include presence of gaps between quad-patches, and retaining critical features. Volume based methods(3D) require TO models to be decomposed to simpler disjoint volumes. While they offer unique advantages over the other two, we are not aware of robust implementations of 3D methods.

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References

[1] Martin P Bendsoe and Ole Sigmund. Topology optimization: Theory, methods and applications, 2003.

[2] Martin P Bendsøe. *Topology optimization.* Springer, 2009.

[3] Ole Sigmund and Kurt Maute. Topology optimization approaches. *Structural and Multidisciplinary Optimization*, 48(6):1031–1055, 2013.

[4] Ji-Hong Zhu, Wei-Hong Zhang, and Liang Xia. Topology optimization in aircraft and aerospace structures design. *Archives of Computational Methods in Engineering*, 23(4):595–622, 2016.

[5] Ji-Hong Zhu, Xiao-Jun Gu, Wei-Hong Zhang, and Pierre Beckers. Structural design of aircraft skin stretch-forming die using topology optimization. *Journal of Computational and Applied Mathematics*, 246:278–288, 2013.

[6] Peter D Dunning, Bret K Stanford, and H Alicia Kim. Coupled aerostructural topology optimization using a level set method for 3d aircraft wings. *Structural and Multidisciplinary Optimization*, 51(5):1113–1132, 2015.

[7] David Walker and David Liu. Topology optimization of an aircraft wing. In *56th AIAA/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference*, page 0976, 2015.

[8] E Kesseler and W. Vankan. Multidisciplinary design analysis and multi-objective optimisation applied to aircraft wing. *WSEAS Transactions on Systems and Control*, 1, 01 2006.

[9] Juan J Alonso, Patrick LeGresley, and Víctor Pereyra. Aircraft design optimization. *Mathematics and Computers in Simulation*, 79(6):1948–1958, 2009.

[10] Chao Li, Il Yong Kim, and Jack Jeswiet. Conceptual and detailed design of an automotive engine cradle by using topology, shape, and size optimization. *Structural and Multidisciplinary Optimization*, 51(2):547–564, 2015.

[11] Denghong Xiao, Xiandong Liu, Wenhua Du, Junyuan Wang, and Tian He. Application of topology optimization to design an electric bicycle main frame. *Structural and Multidisciplinary Optimization*, 46(6):913–929, 2012.

[12] Ching-Hung Chuang, Shikui Chen, Ren-Jye Yang, and Panagiotis Vogiatzis. Topology optimization with additive manufacturing consideration for vehicle load path development. *International Journal for Numerical Methods in Engineering*, 113(8):1434–1445, 2018.

[13] Jun Wu, Niels Aage, Rüdiger Westermann, and Ole Sigmund. Infill optimization for additive manufacturing—approaching bone-like porous structures. *IEEE transactions on visualization and computer graphics*, 24(2):1127–1140, 2017.

[14] Alok Sutradhar, Jaejong Park, Diana Carrau, Tam H Nguyen, Michael J Miller, and Glaucio H Paulino. Designing patient-specific 3d printed craniofacial implants using a novel topology optimization method. *Medical & biological engineering & computing*, 54(7):1123–1135, 2016.

[15] Pedro G Coelho, Scott J Hollister, Colleen L Flanagan, and Paulo R Fernandes. Bioresorbable scaffolds for bone tissue engineering: optimal design, fabrication, mechanical testing and scale-size effects analysis. *Medical engineering & physics*, 37(3):287–296, 2015.

[16] Xiaojian Wang, Shanqing Xu, Shiwei Zhou, Wei Xu, Martin Leary, Peter Choong, M Qian, Milan Brandt, and Yi Min Xie. Topological design and additive manufacturing of porous metals for bone scaffolds and orthopaedic implants: A review. *Biomaterials*, 83:127–141, 2016.

[17] Jaejong Park, Alok Sutradhar, Jami J Shah, and Glaucio H Paulino. Design of complex bone internal structure using topology optimization with perimeter control. *Computers in biology and medicine*, 94:74–84, 2018.

[18] Hongfeng Ma, Jixin Wang, Yongneng Lu, and Youwei Guo. Lightweight design of turnover frame of bridge detection vehicle using topology and thickness optimization. *Structural and Multidisciplinary Optimization*, 59(3):1007–1019, 2019.

[19] Junho Chun, Junho Song, and Glaucio H Paulino. Structural topology optimization under constraints on instantaneous failure probability. *Structural and Multidisciplinary Optimization*, 53(4):773–799, 2016.

[20] Maggie Kociecki and Hojjat Adeli. Shape optimization of free-form steel space-frame roof structures with complex geometries using evolutionary computing. *Engineering Applications of Artificial Intelligence*, 38:168–182, 2015.

[21] B Briseghella, Luigi Fenu, C Lan, E Mazzarolo, and T Zordan. Application of topological optimization to bridge design. *Journal of Bridge Engineering*, 18(8):790–800, 2012.

[22] Zhi Hao Zuo, Xiaodong Huang, Tim Black, Peter Felicetti, et al. Application of topological optimisation technology to bridge design. *Structural Engineering International*, 24(2):185–191, 2014.

[23] Mingdong Zhou, Boyan S Lazarov, Fengwen Wang, and Ole Sigmund. Minimum length scale in topology optimization by geometric constraints. *Computer Methods in Applied Mechanics and Engineering*, 293:266–282, 2015.

[24] Anirudh Krishnakumar and Krishnan Suresh. Hinge-free compliant mechanism design via the topological level-set. *Journal of Mechanical Design*, 137(3):031406, 2015.

[25] Sheng Chu, Liang Gao, Mi Xiao, Zhen Luo, and Hao Li. Stress-based multi-material topology optimization of compliant mechanisms. *International Journal for Numerical Methods in Engineering*, 113(7):1021–1044, 2018.

[26] Qi Xia and Tielin Shi. Topology optimization of compliant mechanism and its support through a level set method. *Computer Methods in Applied Mechanics and Engineering*, 305:359–375, 2016.

[27] David S Makhija and Philip S Beran. Concurrent shape and topology optimization for steady conjugate heat transfer. *Structural and Multidisciplinary Optimization*, 59(3):919–940, 2019.

[28] Shuhao Wu, Yongcun Zhang, and Shutian Liu. Topology optimization for minimizing the maximum temperature of transient heat conduction structure. *Structural and Multidisciplinary Optimization*, pages 1–14, 2019.

[29] Kentaro Yaji, Takayuki Yamada, Masato Yoshino, Toshiro Matsumoto, Kazuhiro Izui, and Shinji Nishiwaki. Topology optimization in thermal-fluid flow using the lattice boltzmann method. *Journal of Computational Physics*, 307:355–377, 2016.

[30] Kentaro Yaji, Takayuki Yamada, Seiji Kubo, Kazuhiro Izui, and Shinji Nishiwaki. A topology optimization method for a coupled thermal–fluid problem using level set boundary expressions. *International Journal of Heat and Mass Transfer*, 81:878–888, 2015.

[31] Jan HK Haertel and Gregory F Nellis. A fully developed flow thermofluid model for topology optimization of 3d-printed air-cooled heat exchangers. *Applied Thermal Engineering*, 119:10–24, 2017.

[32] Peter Coffin and Kurt Maute. Level set topology optimization of cooling and heating devices using a simplified convection model. *Structural and multidisciplinary optimization*, 53(5):985–1003, 2016.

[33] Xiaoping Qian and Ercan M Dede. Topology optimization of a coupled thermal-fluid system under a tangential thermal gradient constraint. *Structural and Multidisciplinary Optimization*, 54(3):531–551, 2016.

[34] Ercan M Dede, Shailesh N Joshi, and Feng Zhou. Topology optimization, additive layer manufacturing, and experimental testing of an air-cooled heat sink. *Journal of Mechanical Design*, 137(11):111403, 2015.

[35] Michael Yu Wang, Xiaoming Wang, and Dongming Guo. A level set method for structural topology optimization. *Computer methods in applied mechanics and engineering*, 192(1-2):227–246, 2003.

[36] Xiaodong Huang and Mike Xie. *Evolutionary topology optimization of continuum structures: methods and applications*. John Wiley & Sons, 2010.

[37] OM Querin, GP Steven, and YM Xie. Evolutionary structural optimisation using a bidirectional algorithm, engrg, 1998.

[38] Hae Chang Gea. Topology optimization: a new microstructure-based design domain method. *Computers & structures*, 61(5):781–788, 1996.

[39] Krishnan Suresh. A 199-line matlab code for pareto-optimal tracing in topology optimization. *Structural and Multidisciplinary Optimization*, 42(5):665–679, 2010.

[40] Krishnan Suresh and Meisam Takalloozadeh. Stress-constrained topology optimization: a topological level-set approach. *Structural and Multidisciplinary Optimization*, 48(2):295–309, 2013.

[41] Martin Philip Bendsøe and Noboru Kikuchi. Generating optimal topologies in structural design using a homogenization method. *Computer methods in applied mechanics and engineering*, 71(2):197–224, 1988.

[42] Ashok V Kumar and Anand Parthasarathy. Topology optimization using b-spline finite elements. *Structural and Multidisciplinary Optimization*, 44(4):471, 2011.

[43] Tej Kumar and Krishnan Suresh. A density-and-strain-based k-clustering approach to microstructural topology optimization. *Structural and Multidisciplinary Optimization*, pages 1–17, 2019.

[44] Alireza H Taheri and Krishnan Suresh. An isogeometric approach to topology optimization of multi-material and functionally graded structures. *International Journal for Numerical Methods in Engineering*, 109(5):668–696, 2017.

[45] M Bartsch, Thomas Weiland, and M Witting. Generation of 3d isosurfaces by means of the marching cube algorithm. *IEEE transactions on magnetics*, 32(3):1469–1472, 1996.

[46] Topology Optimization Roundtable 2019, NM. `http://paulino.ce.gatech.edu/TopOpt%20Workshop%20Website/challenge.php`. Last Accessed: 2019-11-21.

[47] Krishnan Suresh. Topology optimization on the cloud: a confluence of technologies. In *Proceedings of the ASME 2015 International Design Engineering Technical Conferences & Computers and Information in Engineering Conference*, 2015.

[48] Shiguang Deng and Krishnan Suresh. Multi-constrained 3d topology optimization via augmented topological level-set. *Computers & Structures*, 170:1–12, 2016.

[49] Lothar harzheim and Gerhard Graf. A review of optimization of cast parts using topology optimization. *Structural and Multidisciplinary Optimization*, 30(6):491–497, 2005.

[50] Lothar Harzheim and Gerhard Graf. A review of optimization of cast parts using topology optimization. *Structural and multidisciplinary optimization*, 31(5):388–399, 2006.

[51] Jikai Liu and Yongsheng Ma. A survey of manufacturing oriented topology optimization methods. *Advances in Engineering Software*, 100:161–175, 2016.

[52] Kong-Tian Zuo, Li-Ping Chen, Yun-Qing Zhang, and Jingzhou Yang. Manufacturing-and machining-based topology optimization. *The international journal of advanced manufacturing technology*, 27(5-6):531–536, 2006.

[53] Hao Li, Peigen Li, Liang Gao, Li Zhang, and Tao Wu. A level set method for topological shape optimization of 3d structures with extrusion constraints. *Computer Methods in Applied Mechanics and Engineering*, 283:615–635, 2015.

[54] Yang Liu, Zuyu Li, Peng Wei, and Wenwen Wang. Parameterized level-set based topology optimization method considering symmetry and pattern repetition constraints. *Computer Methods in Applied Mechanics and Engineering*, 340:1079–1101, 2018.

[55] Quhao Li, Wenjiong Chen, Shutian Liu, and Huiru Fan. Topology optimization design of cast parts based on virtual temperature method. *Computer-Aided Design*, 94:28–40, 2018.

[56] Sandro L Vatanabe, Tiago N Lippi, Cícero R de Lima, Glaucio H Paulino, and Emílio CN Silva. Topology optimization with manufacturing constraints: A unified projection-based approach. *Advances in Engineering Software*, 100:97–112, 2016.

[57] Jikai Liu and Y-S Ma. 3d level-set topology optimization: a machining feature-based approach. *Structural and Multidisciplinary Optimization*, 52(3):563–582, 2015.

[58] Jeroen P Groen and Ole Sigmund. Homogenization-based topology optimization for high-resolution manufacturable microstructures. *International Journal for Numerical Methods in Engineering*, 113(8):1148–1163, 2018.

[59] Amir M Mirzendehdel, Morad Behandish, and Saigopal Nelaturi. Exploring feasible design spaces for heterogeneous constraints. *Computer-Aided Design*, 115:323–347, 2019.

[60] Wei Gao, Yunbo Zhang, Devarajan Ramanujan, Karthik Ramani, Yong Chen, Christopher B Williams, Charlie CL Wang, Yung C Shin, Song Zhang, and Pablo D Zavattieri. The status, challenges, and future of additive manufacturing in engineering. *Computer-Aided Design*, 69:65–89, 2015.

[61] Tomás Zegard and Glaucio H Paulino. Bridging topology optimization and additive manufacturing. *Structural and Multidisciplinary Optimization*, 53(1):175–192, 2016.

[62] Pierre-Thomas Doutre, Elodie Morretton, Thanh Hoang Vo, Philippe Marin, Franck Pourroy, Guy Prudhomme, and Frederic Vignat. Comparison of some approaches to define a cad model from topological optimization in design for additive manufacturing. In *Advances on Mechanics, Design Engineering and Manufacturing*, pages 233–240. Springer, 2017.

[63] Jikai Liu and Albert C To. Topology optimization for hybrid additive-subtractive manufacturing. *Structural and Multidisciplinary Optimization*, 55(4):1281–1299, 2017.

[64] Martin Leary, Luigi Merli, Federico Torti, Maciej Mazur, and Milan Brandt. Optimal topology for additive manufacture: A method for enabling additive manufacture of support-free optimal structures. *Materials & Design*, 63:678–690, 2014.

[65] Amir M Mirzendehdel and Krishnan Suresh. Support structure constrained topology optimization for additive manufacturing. *Computer-Aided Design*, 81:1–13, 2016.

[66] Yoram Mass and Oded Amir. Topology optimization for additive manufacturing: Accounting for overhang limitations using a virtual skeleton. *Additive Manufacturing*, 18:58–73, 2017.

[67] Alain Garaigordobil, Rubén Ansola, Javier Santamaría, and Igor Fernández de Bustos. A new overhang constraint for topology optimization of self-supporting structures in additive manufacturing. *Structural and Multidisciplinary Optimization*, 58(5):2003–2017, 2018.

[68] Alain Garaigordobil, Rubén Ansola, Estrella Veguería, and Igor Fernandez. Overhang constraint for topology optimization of self-supported compliant mechanisms considering additive manufacturing. *Computer-Aided Design*, 109:33–48, 2019.

[69] Tsz-Ho Kwok, Yongqiang Li, and Yong Chen. A structural topology design method based on principal stress line. *Computer-Aided Design*, 80:19–31, 2016.

[70] Kunal Mhapsekar, Matthew McConaha, and Sam Anand. Additive manufacturing constraints in topology optimization for improved manufacturability. *Journal of Manufacturing Science and Engineering*, 140(5):051017, 2018.

[71] Xiaoping Qian. Undercut and overhang angle control in topology optimization: a density gradient based integral approach. *International Journal for Numerical Methods in Engineering*, 111(3):247–272, 2017.

[72] Francesco Mezzadri, Vladimir Bouriakov, and Xiaoping Qian. Topology optimization of self-supporting support structures for additive manufacturing. *Additive Manufacturing*, 21:666–682, 2018.

[73] Matthijs Langelaar. Topology optimization of 3d self-supporting structures for additive manufacturing. *Additive Manufacturing*, 12:60–70, 2016.

[74] Aaditya Chandrasekhar, Tej Kumar, and Krishnan Suresh. Build optimization of fiber-reinforced additively manufactured components. *Structural and Multidisciplinary Optimization*, Jul 2019.

[75] John C Steuben, Athanasios P Iliopoulos, and John G Michopoulos. Multiscale topology optimization for additively manufactured objects. *Journal of Computing and Information Science in Engineering*, 18(3):031002, 2018.

[76] Jikai Liu, Andrew T Gaynor, Shikui Chen, Zhan Kang, Krishnan Suresh, Akihiro Takezawa, Lei Li, Junji Kato, Jinyuan Tang, Charlie CL Wang, et al. Current and future trends in topology optimization for additive manufacturing. *Structural and Multidisciplinary Optimization*, 57(6):2457–2483, 2018.

[77] Martin Philip Bendsøe and Helder Carrico Rodrigues. Integrated topology and boundary shape optimization of 2-d solids. *Computer Methods in Applied Mechanics and Engineering*, 87(1):15–34, 1991.

[78] Niels Olhoff, Martin P Bendsøe, and John Rasmussen. On cad-integrated structural topology and design optimization. *Computer Methods in Applied Mechanics and Engineering*, 89(1-3):259–279, 1991.

[79] Mingdong Zhou and Michael Yu Wang. Engineering feature design for level set based structural optimization. *Computer-Aided Design*, 45(12):1524–1537, 2013.

[80] Jiaqin Chen, Vadim Shapiro, Krishnan Suresh, and Igor Tsukanov. Shape optimization with topological changes and parametric control. *International journal for numerical methods in engineering*, 71(3):313–346, 2007.

[81] Poh-Soong Tang and Kuang-Hua Chang. Integration of topology and shape optimization for design of structural components. *Structural and Multidisciplinary Optimization*, 22(1):65–82, 2001.

[82] C-Y Lin and L-S Chao. Automated image interpretation for integrated topology and shape optimization. *Structural and Multidisciplinary Optimization*, 20(2):125–137, 2000.

[83] Yunbo Zhang and Tsz-Ho Kwok. Customization and topology optimization of compression casts/braces on two-manifold surfaces. *Computer-Aided Design*, 111:113–122, 2019.

[84] Asger Nyman Christiansen, J Andreas Bærentzen, Morten Nobel-Jørgensen, Niels Aage, and Ole Sigmund. Combined shape and topology optimization of 3d structures. *Computers & Graphics*, 46:25–35, 2015.

[85] Xu Guo, Weisheng Zhang, and Wenliang Zhong. Topology optimization based on moving deformable components: A new computational framework. *arXiv preprint arXiv:1404.4820*, 2014.

[86] Weisheng Zhang, Jie Yuan, Jian Zhang, and Xu Guo. A new topology optimization approach based on moving morphable components (mmc) and the ersatz material model. *Structural and Multidisciplinary Optimization*, 53(6):1243–1260, 2016.

[87] Weisheng Zhang, Dong Li, Jie Yuan, Junfu Song, and Xu Guo. A new three-dimensional topology optimization method based on moving morphable components (mmcs). *Computational Mechanics*, 59(4):647–665, 2017.

[88] Bryan Bell, Julian Norato, and Daniel Tortorelli. A geometry projection method for continuum-based topology optimization of structures. In *12th AIAA Aviation Technology, integration, and operations (ATIO) conference and 14th AIAA/ISSMO multidisciplinary analysis and optimization conference*, page 5485, 2012.

[89] JA Norato, BK Bell, and Daniel A Tortorelli. A geometry projection method for continuum-based topology optimization with discrete elements. *Computer Methods in Applied Mechanics and Engineering*, 293:306–327, 2015.

[90] Hung-Yun Lin, Mahidhar Rayasam, and Ganesh Subbarayan. Isocomp: Unified geometric and material composition for optimal topology design. *Structural and Multidisciplinary Optimization*, 51(3):687–703, 2015.

[91] Yuliang Gao, Yujie Guo, and Shijie Zheng. A nurbs-based finite cell method for structural topology optimization under geometric constraints. *Computer Aided Geometric Design*, 72:1–18, 2019.

[92] Weihong Zhang, Linying Zhao, Tong Gao, and Shouyu Cai. Topology optimization with closed B-splines and Boolean operations. *Computer Methods in Applied Mechanics and Engineering*, 315:652–670, 2017.

[93] Julián A Norato. Topology optimization with supershapes. *Structural and Multidisciplinary Optimization*, 58(2):415–434, 2018.

[94] Daicong Da, Liang Xia, Guangyao Li, and Xiaodong Huang. Evolutionary topology optimization of continuum structures with smooth boundary representation. *Structural and Multidisciplinary Optimization*, 57(6):2143–2159, 2018.

[95] Hassan A Jahangiry and S Mehdi Tavakkoli. An isogeometrical approach to structural level set topology optimization. *Computer Methods in Applied Mechanics and Engineering*, 319:240–257, 2017.

[96] Pilseong Kang and Sung-Kie Youn. Isogeometric topology optimization of shell structures using trimmed nurbs surfaces. *Finite Elements in Analysis and Design*, 120:18–40, 2016.

[97] Yu-Deok Seo, Hyun-Jung Kim, and Sung-Kie Youn. Isogeometric topology optimization using trimmed spline surfaces. *Computer Methods in Applied Mechanics and Engineering*, 199(49-52):3270–3296, 2010.

[98] Yundong Gai, Xuefeng Zhu, Yongjie Jessica Zhang, Wenbin Hou, and Ping Hu. Explicit isogeometric topology optimization based on moving morphable voids with closed b-spline boundary curves. *Structural and Multidisciplinary Optimization*, pages 1–20, 2019.

[99] Jie Gao, Liang Gao, Zhen Luo, and Peigen Li. Isogeometric topology optimization for continuum structures using density distribution function. *International Journal for Numerical Methods in Engineering*, 119(10):991–1017, 2019.

[100] Ivan Sosnovik and Ivan Oseledets. Neural networks for topology optimization. *Russian Journal of Numerical Analysis and Mathematical Modelling*, 34(4):215–223, 2019.

[101] M-H Herman Shen and Liang Chen. A new cgan technique for constrained topology design optimization. *arXiv preprint arXiv:1901.07675*, 2019.

[102] Sharad Rawat and MH Shen. A novel topology design approach using an integrated deep learning network architecture. *arXiv preprint arXiv:1808.02334*, 2018.

[103] Sharad Rawat and MH Herman Shen. Application of adversarial networks for 3d structural topology optimization. Technical report, SAE Technical Paper, 2019.

[104] Xin Lei, Chang Liu, Zongliang Du, Weisheng Zhang, and Xu Guo. Machine learning-driven real-time topology optimization under moving morphable component-based framework. *Journal of Applied Mechanics*, 86(1), 2019.

[105] Remondino Fabio et al. From point cloud to surface: the modeling and visualization problem. *International Archives of Photogrammetry, Remote Sensing and Spatial Information Sciences*, 34(5):W10, 2003.

[106] Atul Thakur, Ashis Gopal Banerjee, and Satyandra K Gupta. A survey of cad model simplification techniques for physics-based simulation applications. *Computer-Aided Design*, 41(2):65–80, 2009.

[107] Pierre Alliez, Giuliana Ucelli, Craig Gotsman, and Marco Attene. Recent advances in remeshing of surfaces. In *Shape analysis and structuring*, pages 53–82. Springer, 2008.

[108] Daniel Sieger and Mario Botsch. The polygon mesh processing library, 2019. http://www.pmp-library.org.

[109] Wenzel Jakob, Marco Tarini, Daniele Panozzo, and Olga Sorkine-Hornung. Instant field-aligned meshes. *ACM Transactions on Graphics (Proceedings of SIGGRAPH ASIA)*, 34(6), November 2015.

[110] Michael Kazhdan, Matthew Bolitho, and Hugues Hoppe. Poisson surface reconstruction. In *Proceedings of the Fourth Eurographics Symposium on Geometry Processing*, SGP '06, pages 61–70, Aire-la-Ville, Switzerland, Switzerland, 2006. Eurographics Association.

[111] Marco Attene, Bianca Falcidieno, Jarek Rossignac, and Michela Spagnuolo. Edge-sharpener: Recovering sharp features in triangulations of non-adaptively re-meshed surfaces. In *Proceedings of the 2003 Eurographics/ACM SIGGRAPH Symposium on Geometry Processing*, SGP '03, pages 62–69, Aire-la-Ville, Switzerland, Switzerland, 2003. Eurographics Association.

[112] Gregory M. Nielson. Dual marching cubes. In *Proceedings of the Conference on Visualization '04*, VIS '04, pages 489–496, Washington, DC, USA, 2004. IEEE Computer Society.

[113] Thomas Lewiner, Hélio Lopes, Antônio Wilson Vieira, and Geovan Tavares. Efficient implementation of marching cubes' cases with topological guarantees. *Journal of Graphics Tools*, 8(2):1–15, 2003.

[114] Jonathan Richard Shewchuk. Tetrahedral mesh generation by delaunay refinement. In *Symposium on Computational Geometry*, pages 86–95, 1998.

[115] Hang Si. Tetgen, a delaunay-based quality tetrahedral mesh generator. *ACM Transactions on Mathematical Software (TOMS)*, 41(2):11, 2015.

[116] Tamal K Dey and Samrat Goswami. Tight cocone: a water-tight surface reconstructor. In *Proceedings of the eighth ACM symposium on Solid modeling and applications*, pages 127–134. ACM, 2003.

[117] Siu-Wing Cheng, Tamal K. Dey, and Edgar A. Ramos. Delaunay refinement for piecewise smooth complexes. *Discrete & Computational Geometry*, 43(1):121–166, Jan 2010.

[118] Liangliang Nan and Peter Wonka. Polyfit: Polygonal surface reconstruction from point clouds. In *Proceedings of the IEEE International Conference on Computer Vision*, pages 2353–2361, 2017.

[119] Guilian Yi and Nam H Kim. Identifying boundaries of topology optimization results using basic parametric features. *Structural and Multidisciplinary Optimization*, 55(5):1641–1654, 2017.

[120] Yangyan Li, Xiaokun Wu, Yiorgos Chrysanthou, Andrei Sharf, Daniel Cohen-Or, and Niloy J. Mitra. Globfit: Consistently fitting primitives by discovering global relations. *ACM Transactions on Graphics*, 30(4):52:1–52:12, 2011.

[121] Ruwen Schnabel, Roland Wahl, and Reinhard Klein. Efficient ransac for point-cloud shape detection. *Computer Graphics Forum*, 26(2):214–226, June 2007.

[122] Saumitra Joshi, Juan Carlos Medina, Erik Wannerberg, Anna Yurova, Severin Reiz, and Benjamin Rüth. Cad-integrated topology optimization. *BGCE Honor. Proj. Report, Bavar. Grad. Sch. Comput. Eng.*, 2016.

[123] Shutian Liu, Quhao Li, Junhuan Liu, Wenjiong Chen, and Yongcun Zhang. A realization method for transforming a topology optimization design into additive manufacturing structures. *Engineering*, 4(2):277–285, 2018.

[124] JM Chacón, JC Bellido, and A Donoso. Integration of topology optimized designs into cad/cam via an iges translator. *Structural and Multidisciplinary Optimization*, 50(6):1115–1125, 2014.

[125] Atsushi Koguchi and Noboru Kikuchi. A surface reconstruction algorithm for topology optimization. *Engineering with Computers*, 22(1):1–10, 2006.

[126] AL Marsan and D Dutta. Construction of a surface model and layered manufacturing data from 3d homogenization output. *Journal of Mechanical Design*, 118(3):412–418, 1996.

[127] Yosef M Yoely, Oded Amir, and Iddo Hanniel. Topology and shape optimization with explicit geometric constraints using a spline-based representation and a fixed grid. *Procedia Manufacturing*, 21:189–196, 2018.

[128] Wei Zhao, Shuming Gao, and Hongwei Lin. A robust hole-filling algorithm for triangular mesh. *Vis. Comput.*, 23(12):987–997, November 2007.

[129] John Branch, Flavio Prieto, and Pierre Boulanger. A hole-filling algorithm for triangular meshes using local radial basis function. In Philippe P. Pébay, editor, *Proceedings of the 15th International Meshing Roundtable*, pages 411–431, Berlin, Heidelberg, 2006. Springer Berlin Heidelberg.

[130] Brian Curless and Marc Levoy. A volumetric method for building complex models from range images. In *Proceedings of the 23rd Annual Conference on Computer Graphics and Interactive Techniques*, SIGGRAPH '96, pages 303–312, New York, NY, USA, 1996. ACM.

[131] Peter Liepa. Filling holes in meshes. In *Proceedings of the 2003 Eurographics/ACM SIGGRAPH Symposium on Geometry Processing*, SGP '03, pages 200–205, Aire-la-Ville, Switzerland, Switzerland, 2003. Eurographics Association.

[132] Edwin Catmull and James Clark. Recursively generated b-spline surfaces on arbitrary topological meshes. *Computer-aided design*, 10(6):350–355, 1978.

[133] Daniel Doo and Malcolm Sabin. Behaviour of recursive division surfaces near extraordinary points. *Computer-Aided Design*, 10(6):356–360, 1978.

[134] Charles Loop. Smooth subdivision surfaces based on triangles. *Master's thesis, University of Utah, Department of Mathematics*, 1987.

[135] Herbert Edelsbrunner and Daniel R Grayson. Edgewise subdivision of a simplex. *Discrete & Computational Geometry*, 24(4):707–719, 2000.

[136] Martin Marinov, Marco Amagliani, Tristan Barback, Jean Flower, Stephen Barley, Suguru Furuta, Peter Charrot, Iain Henley, Nanda Santhanam, G Thomas Finnigan, et al. Generative design conversion to editable and watertight boundary representation. *Computer-Aided Design*, 115:194–205, 2019.

[137] Thomas W Sederberg, Jianmin Zheng, Almaz Bakenov, and Ahmad Nasri. T-splines and t-nurccs. In *ACM transactions on graphics (TOG)*, volume 22, pages 477–484. ACM, 2003.

[138] Ming-Hsiu Hsu and Yeh-Liang Hsu. Interpreting three-dimensional structural topology optimization results. *Computers & structures*, 83(4-5):327–337, 2005.

[139] Yeh-Liang Hsu, Ming-Sho Hsu, and Chuan-Tang Chen. Interpreting results from topology optimization using density contours. *Computers & Structures*, 79(10):1049–1058, 2001.

[140] Jean-Christophe Cuillière, Vincent Francois, and Jean-Marc Drouet. Towards the integration of topology optimization into the cad process. *Computer-Aided Design and Applications*, 11(2):120–140, 2014.

[141] Jean-Christophe Cuillière, Vincent Francois, and Jean-Marc Drouet. Automatic mesh generation and transformation for topology optimization methods. *Computer-Aided Design*, 45(12):1489–1506, 2013.

[142] Shane Larsen and C Greg Jensen. Converting topology optimization results into parametric cad models. *Computer-Aided Design and Applications*, 6(3):407–418, 2009.

[143] Tao Du, Jeevana Priya Inala, Yewen Pu, Andrew Spielberg, Adriana Schulz, Daniela Rus, Armando Solar-Lezama, and Wojciech Matusik. Inversecsg: Automatic conversion of 3d models to csg trees. *ACM Trans. Graph.*, 37(6):213:1–213:16, December 2018.