GNURBS Lab

An Interactive MATLAB toolbox for GNURBS

Reference: Taheri, A. H., Abolghasemi, S., Suresh, K., "Generalizations of Non-Uniform Rational B-Splines: Theory, Software and Applications", submitted to Advances in Engineering Software, January 2019 (preprint available at www.ersl.wisc.edu).

A. Introduction

This interactive MATLAB toolbox is devised to generate and manipulate GNURBS curves. In classic NURBS, the weights are equal along all physical coordinates. By allowing the weights to change independently in each physical coordinate, a new curve is generated which is called Generalized NURBS (GNURBS). Different features and capabilities of this toolbox are explained in the following sections.

B. Main window

When the code *GNURBS_Lab.m* is run, the main window appears. This window, which is shown in Figure 1, is composed of four main sections:

I. *Left panel*: in this panel, the input parameters are inserted, and different settings and selections can be made. The menus of this panel and their functions will be explained.

II. *Status bar*: in this field, the current status of the figure, the actions that are needed to be done by the user and other useful information about the graph is displayed.

III. *Right panel*: in this panel, different settings and selections can be done. The menus of this panel and their function will be explained

IV. *Main graph*: The curve is shown in this area.

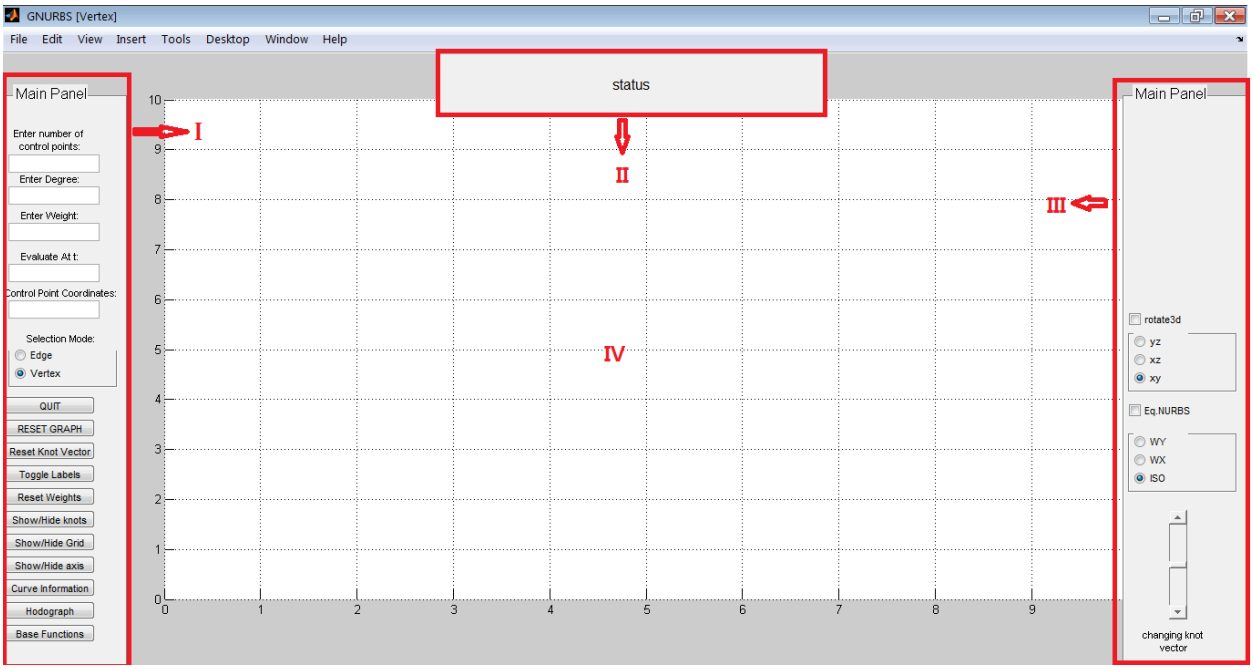

Figure 1: Main window.

The different fields and menus of the right and left panels are numbered in Figure 2. These numbers are referenced in the following sections.

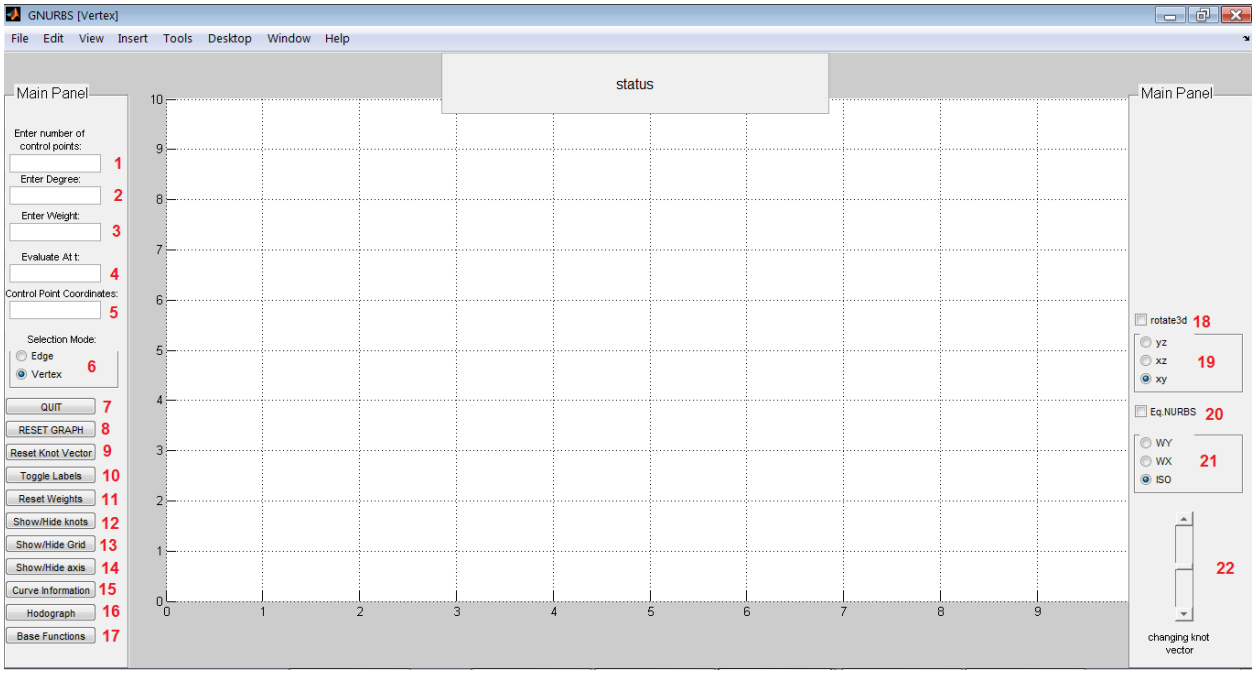

Figure 2: Numbering the different fields and menus of right and left panels.

The toolbox is devised to be user-friendly, and employing most of its features is straightforward. However, a detailed description of its different features is elaborated in the following sections. The following table describes the action of different mouse buttons.

Table 1. The actions of different mouse buttons in GNURBS Lab.

L-Click	*Insert new control point / release a dragged one*
R-Click	*Select control point / edge*
Scroll	*Change weight*
Double R-Click	*Move a control point*

C. Constructing a curve

To construct a curve, first enter the number of control points and the degree of the curve (p) in the corresponding fields in the left panel, (fields **1** and **2** in Figure 2) and then press enter. Next, insert the specified number of control points at desired locations by left clicking. An example of a degree 3 curve with 6 control points is shown in Figure 3.

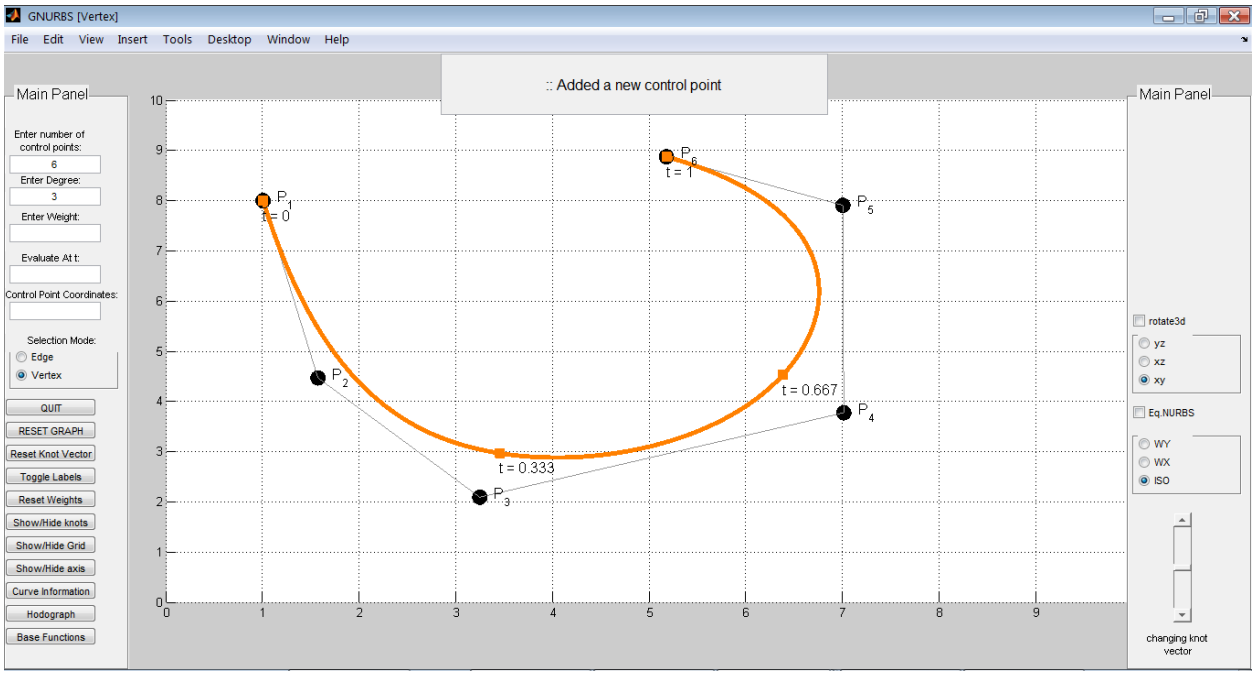

Figure 3: Constructing a degree 3 curve with 6 control points.

Each control point is labeled with P_i. The curve is plotted in orange and the *control polygon* is shown in gray. The control points and sides of control polygon can be selected in "*Vertex*" and "*Edge*" mode, respectively. The knot vector is defined in the interval [0,1], where the multiplicity of the first and last knots is p+1, and the interior knots are spaced equally by default. The knots are also shown in the graph by default, and the corresponding knot values are written beneath each knot. The user can choose to show or hide the knots by clicking the "Show/Hide knots" button (**12**) from the left panel.

The number of control points as well as the degree of the curve can be changed by entering the desired values in fields **1** and **2**.

D. Moving the control points

In order to move a control point, double right click on a control point and move the mouse. By moving the mouse, the selected control point moves and the curve updates. To release, left click or press Escape.

E. Changing the weights

By default, the weights of all control points are set to 1. To change the weight of each control point, first select that control point by right clicking on it (in *Vertex* mode) and then scroll. The weight can be changed in 3 different ways:

ISO: *isoparametric*, i.e. the weight of x and y coordinates change simultaneously, as in classic NURBS. This is the default option.

WX: only the weight of x coordinate changes.

WY: only the weight of y coordinate changes.

As a weight is changed, the curve updates and the value of the new weight is shown in the status bar. As an example, in Figure 4, the ISO weight of P_3 is set to 0.3.

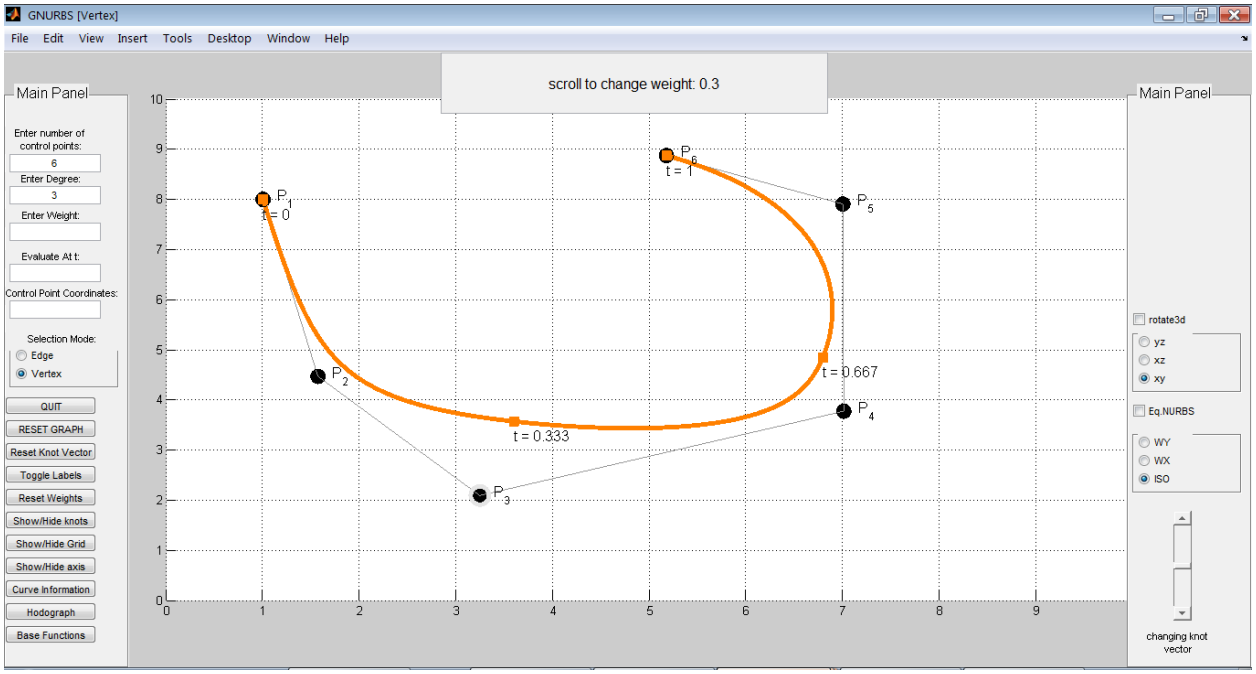

Figure 4: Changing the weight of P_3 to 0.3.

The method of changing the weights can be selected from the right panel (21). For example, to change the weight of P_4 in the x direction, select WX, then select P_4 by right clicking on it and then scroll to update the curve. The value of WX or WY updates in the status field as the user scrolls. In Figure 5, the weight of P_4 in the x direction is set to 8.

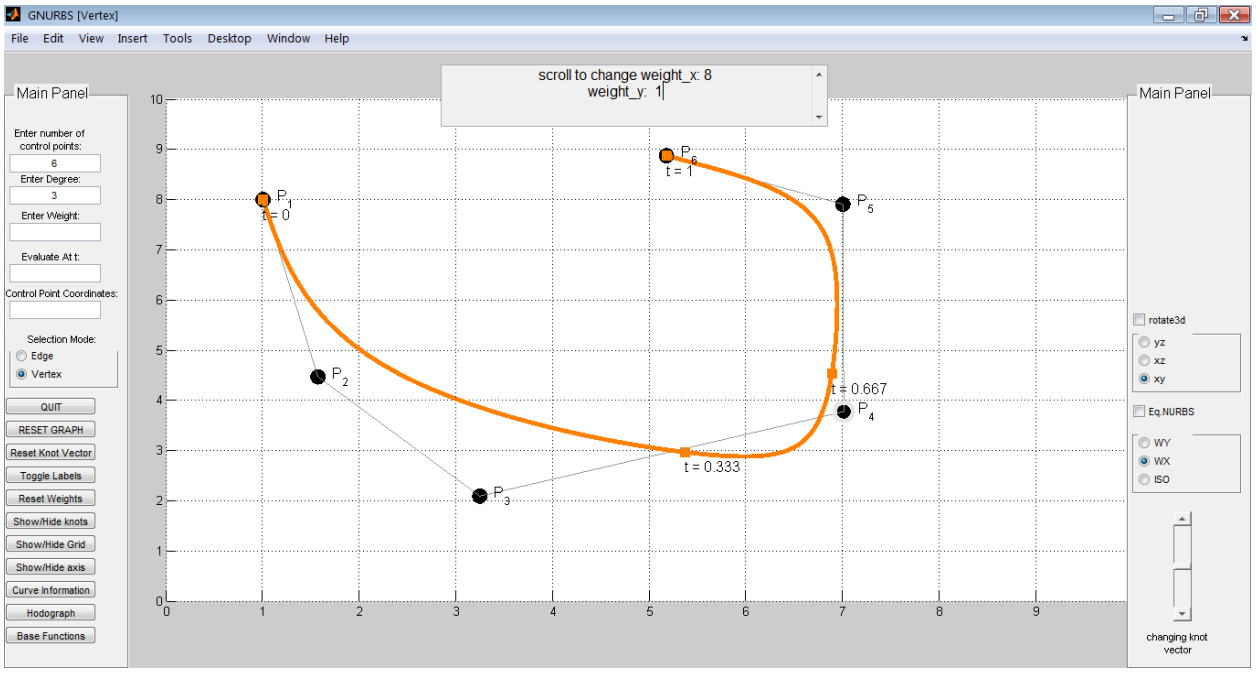

Figure 5: Changing the weight of P_4 in the x direction to 8.

Also, the user can specify the desired exact value of a weight in the "Enter Weight:" field (3) in the left panel. For example, to change the weight of P_2 in the y direction to 10, select WY from right panel (21), then insert 10 in the "Enter Weight" filed and press enter (see Figure 6).

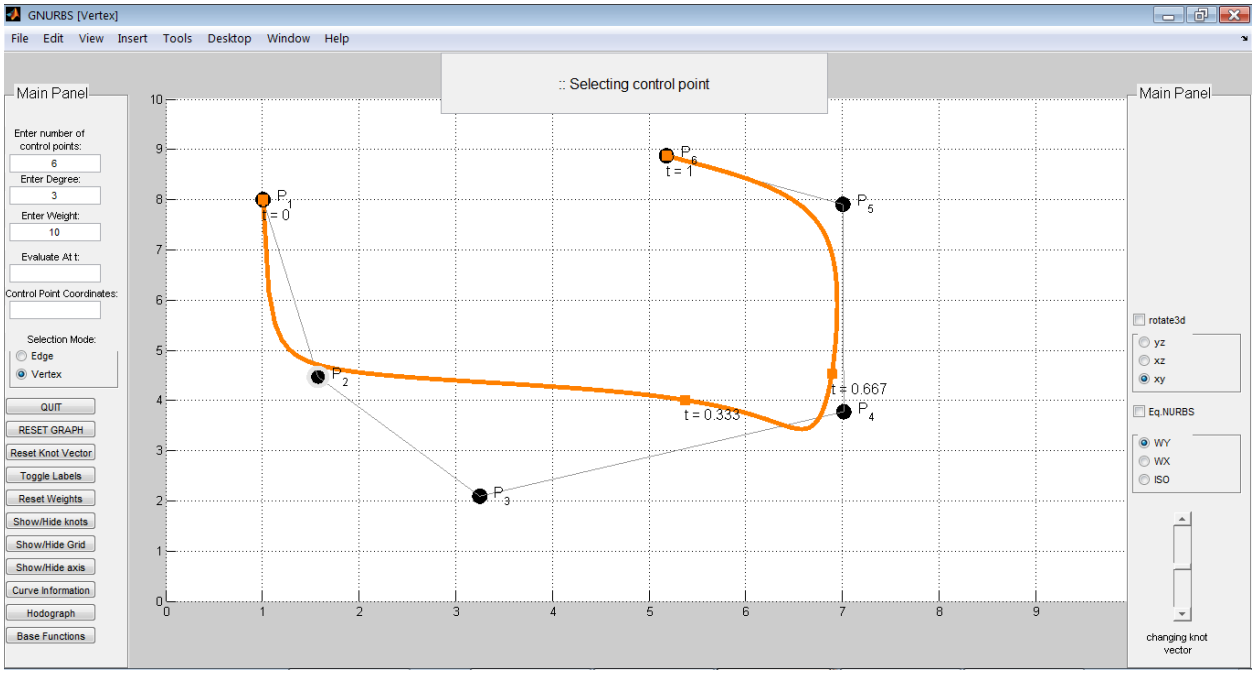

Figure 6: Changing the weight of P_2 in the y direction to 10 from the "Enter weight" field of the left panel.

The user can reset all the weights to 1 by pressing the "Reset Weights" button (11) from the left panel.

F. Plotting the equivalent NURBS curve

It can be proved that "every GNURBS curve of degree *p* and dimension *m* can be exactly transformed into a NURBS curve of degree *m*×*p*" *(see reference paper above).* In order to see the equivalent higher order NURBS curve to a GNURBS curve, check box (20) "Eq. NURBS" in the right panel. In Figure 7, the equivalent curve of Figure 6 is shown. It is noted that in the case of having identical weights in all directions (classic NURBS), checking this box will simply show an order elevated NURBS curve.

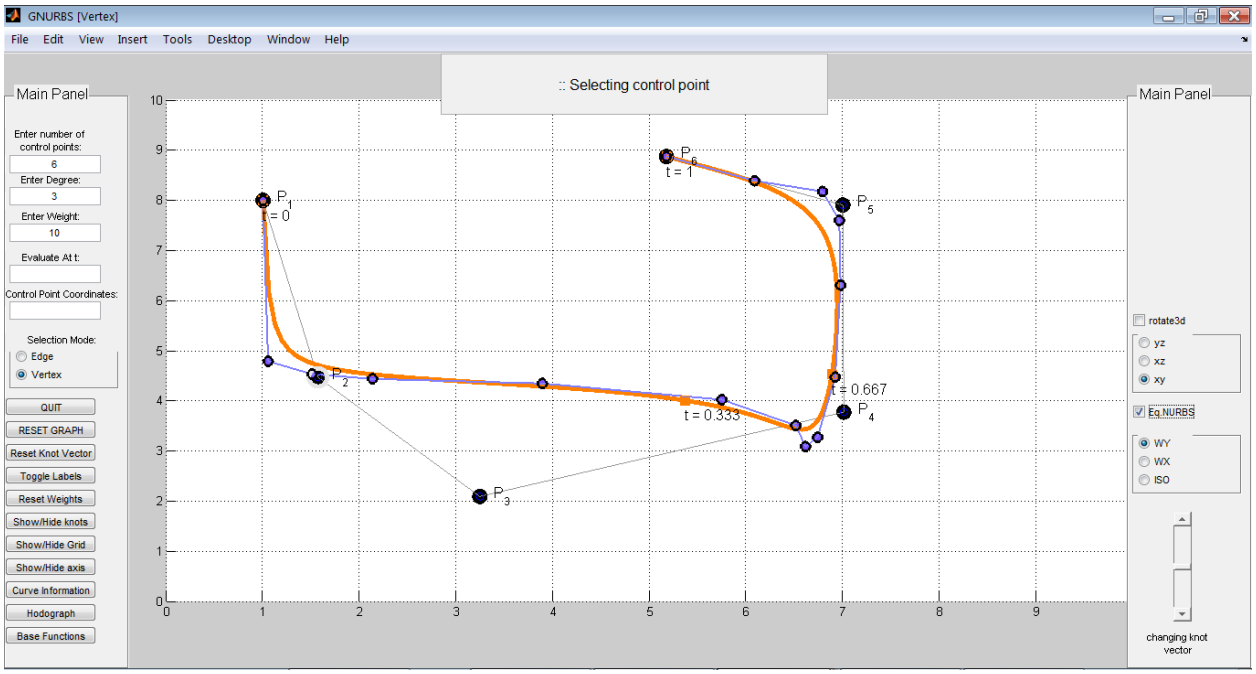

Figure 7: Equivalent higher order NURBS curve of GNURBS curve in Figure 6.

It is noted that as long as the box "Eq. NURBS" is checked, by changing the weight or moving the control points, the equivalent NURBS curve also updates.

G. Evaluating the curve at a certain parameter value *t*

In the field "Evaluate at *t*" of the left panel (4), the user can enter a value for *t* to calculate the corresponding point on the curve. The coordinates of this point will be shown in the status bar and the position of this point on the curve will be shown by a red cross (see Figure 8).

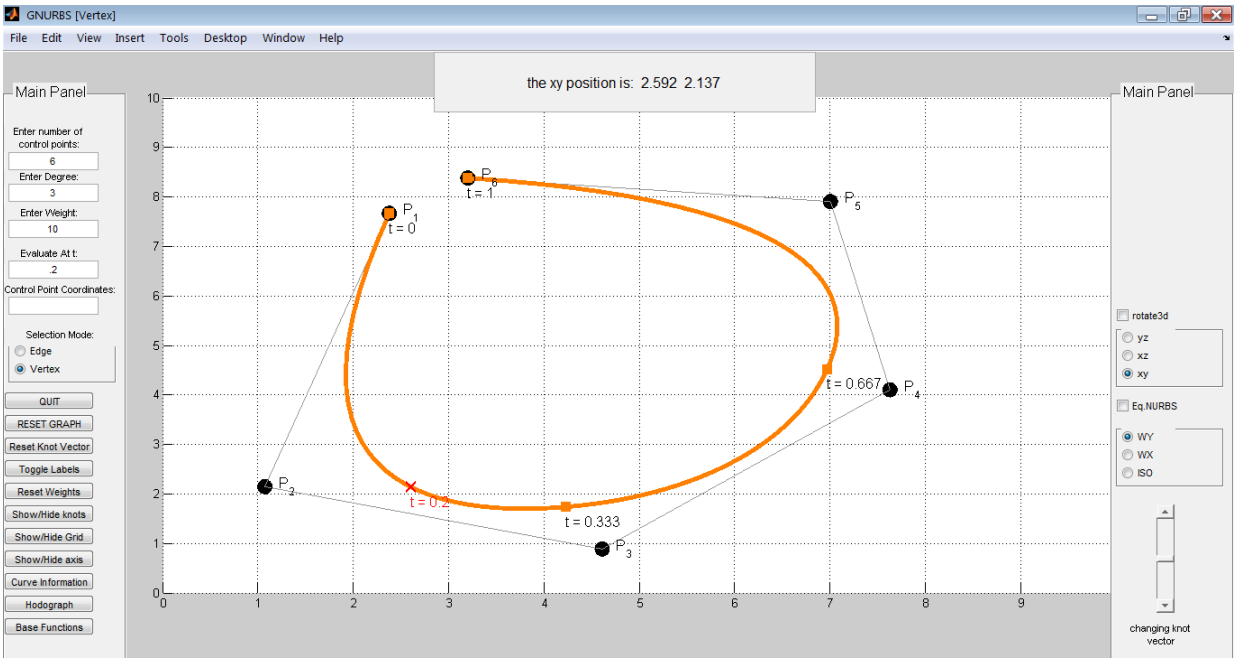

Figure 8: Evaluating the position and coordinates of the point corresponding to t=0.2 on the curve.

H. Specifying exact coordinates for control points

One can also assign exact numerical values to the coordinates of control points. This can be done by selecting a control point and entering its coordinates in the form "*a*,*b*" (where *a* and *b* are the *x* and *y* coordinates of the control point) in field (5) of the left panel "Cont. Point Coords:". For example, in Figure 9, the coordinates of P_4 are changed to (4,4).

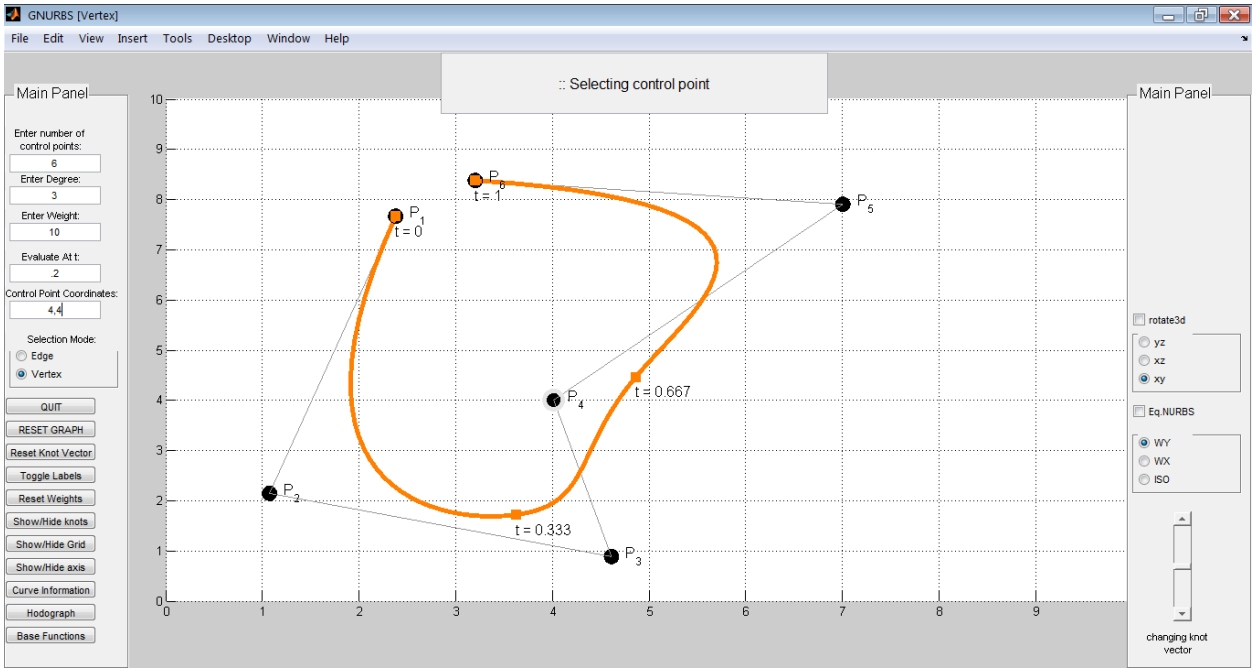

Figure 9: Changing the coordinates of P_4 to (4,4) from the left panel.

I. Changing the knot vector

By default, the selection mode is set to "*Vertex*". To change the knot-vector, first select "*Edge*" from the "Selection Mode" in the left panel (6). Then select an edge by right clicking on it, now move the slider in the right panel (22) up or down to change the knot vector and see how the curve updates. To reset the knot vector to its initial value, press "Reset Knot Vector" button (9) in the left panel.

J. Rotating the graph

The user can rotate the graph in space by checking the "rotate 3d" box in the right panel (18). The cursor changes to a circular arrow. Now by holding the left click and moving the cursor, the plot rotates. Also, the user can choose the view plane of the graph (*xz* or *xy* or *yz*) from the right panel (19).

K. Plotting the Basis functions

By clicking the "Basis Functions" button (17) from the left panel, the main window automatically resizes to show the NURBS curve and the basis functions on the screen. The graph of basis functions is color coded, implying that a color is assigned to each control point and the corresponding basis function is plotted in the same color (see Figure 10). The basis functions graph is also interactive. That is, by selecting a control point and changing the weight, the graph of basis functions also updates. Also, by selecting each weight type from the right panel, the graph of basis functions for the selected weight is depicted and updates by changing the corresponding weight value.

Figure 10: Interactive plot of basis functions corresponding to each control point.

L. Curve Information

The user can see some information about the curve by clicking the "Curve Information" button (15) from the left panel. This information is shown in the status field as demonstrated in Figure 11.

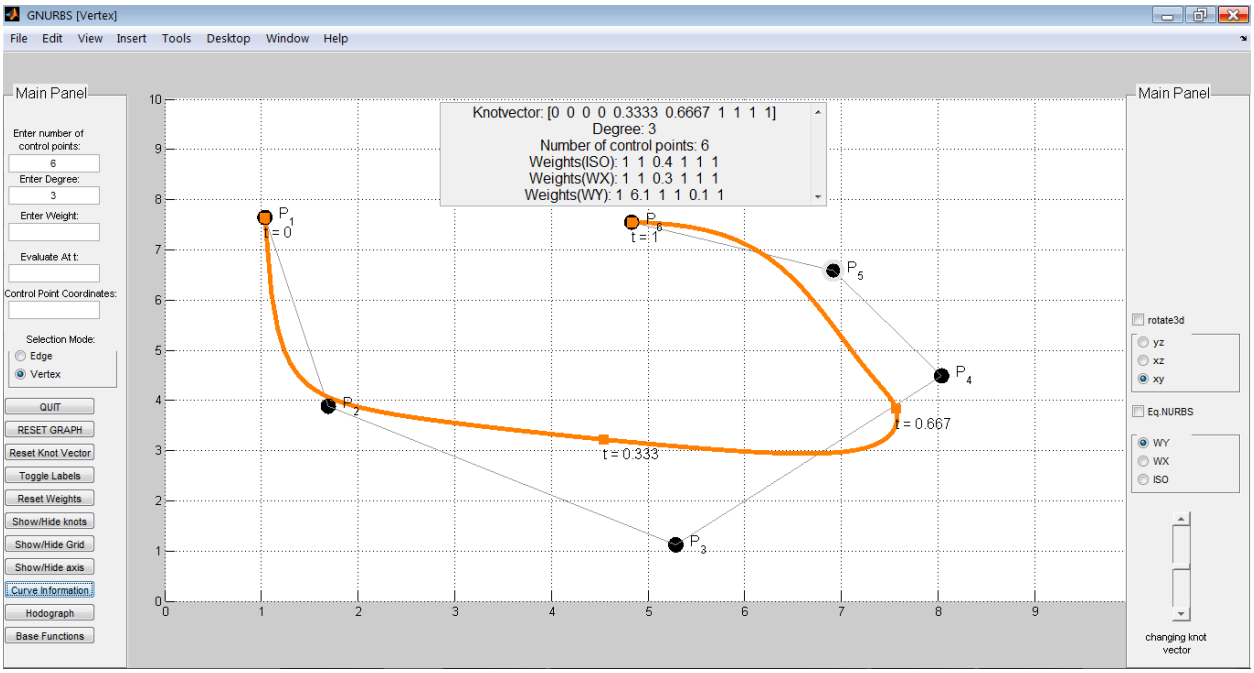

Figure 11: Displaying the curve information by clicking the "Curve information" button from the left panel.

M. Showing different labels

The user can switch between different labels which are shown in the graph by clicking the "Toggle labels" button (**10**) from the left panel several times. The active label is shown in the status bar.

The default label is 'Control Point' labels. By clicking the "Toggle Labels" button, the labels change respectively to: 'Control Point Weights', 'Blossom Labels', 'Knot Spans', 'Control Point Coordinates' and 'No Labels'. These labels are shown in Figure 12.

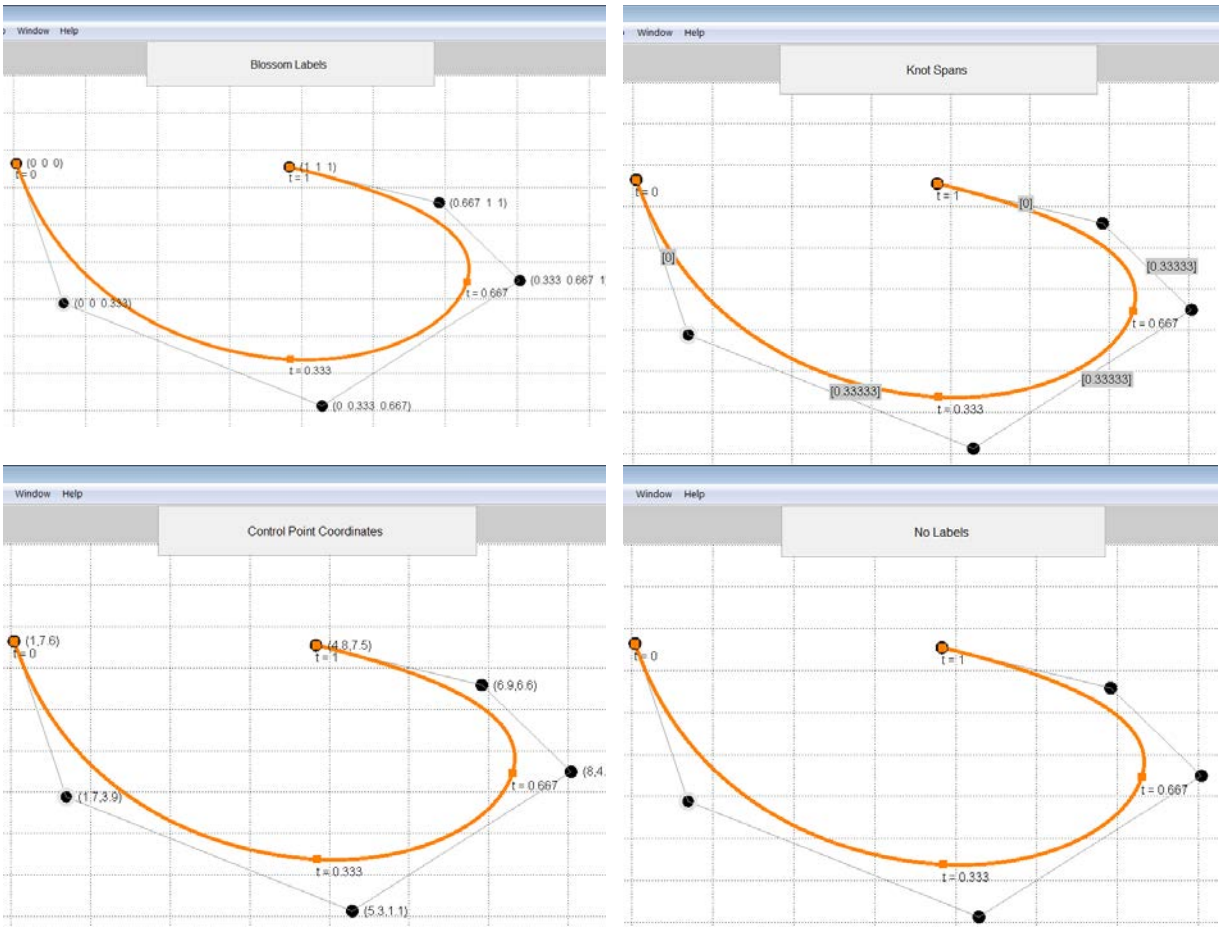

Figure 12: Different labels can be shown by clicking the "Toggle Labels" button several times.

N. Hodograph

The user can also see the Hodograph of a curve by clicking button (16) from the left panel as shown in Figure 13.

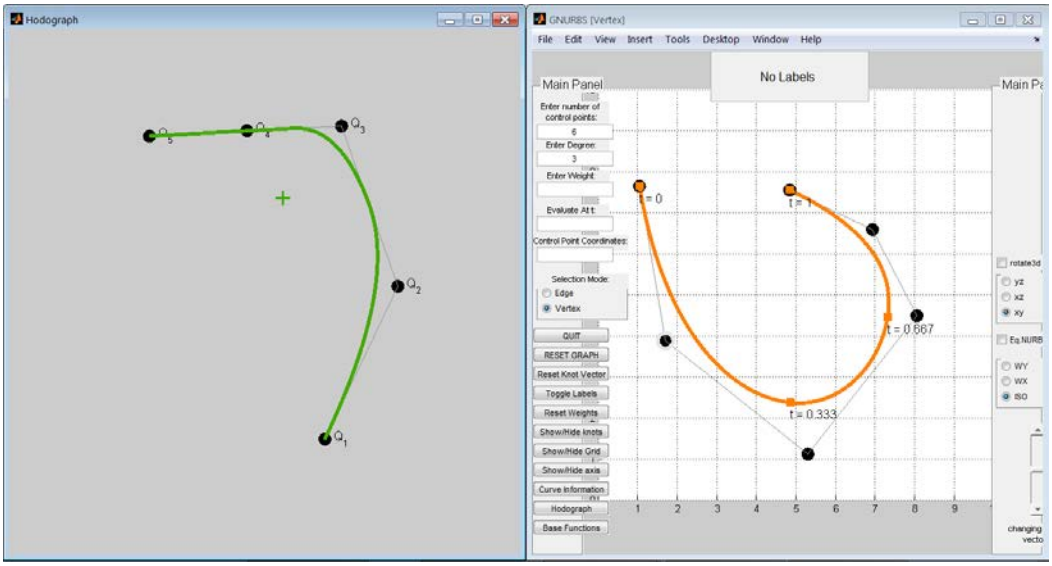

Figure 13: The hodograph of a NURBS curve.